The characteristic properties of waste rock piles in terms of metal leaching

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ABSTRACT

Surface/ground waters could be polluted when rain-water and/or snow-melt water infiltrate through waste rock piles at mine sites and dissolve secondary minerals (salts) from rock surfaces. It is important to reduce solute loading by the optimal configuration of waste rock piles. This requires the proper definition and determination of the characteristic properties of waste rock piles in terms of metal leaching and, in particular, rate control mechanisms and scaling laws, and their dependence upon configuration variables. For revealing these characteristic properties this paper proposes a pile-scale C-Q relation: \( C = C_s(1 - e^{-P/Q}) \), (\( P ≡ k\beta\psi \)), where \( C \) and \( C_s \) are respectively solute concentration and particle’s saturation concentration, \( Q \) is the flow rate of the water through a waste rock pile, \( k \) represents the effective or average dissolution coefficient of a mineral specie from rock surfaces, \( \beta \) represents rock pile depth, \( \lambda \) represents the ratio of the sum of the surface areas of rocks to the volume that the rocks occupy, and \( \psi \) is the sum of the cross-sections of water-flow channels in a waste rock pile. The two characteristic properties revealed by the C-Q relation are: (1) \( P \), the product of \( k, \beta, \lambda \), and \( \psi \) (\( P ≡ k\beta\psi \)), which is the characteristic property of a waste rock pile in terms of metal leaching, named here the solute production potential; and (2) the ratio of \( P \) to \( Q \), \( P/Q \), a non-dimensional number, designated as \( \alpha \) (\( \alpha ≡ P/Q \)), which is the scaling law and the rate control mechanism indicator. The value of \( \alpha \) quantitatively indicates what controls the rate of mineral dissolution, and it also relates smaller-scale metal-leaching testing results to their corresponding full scales. When \( \alpha \) becomes small, say \( \alpha < 0.5 \), the rate of solute production potential \( P \) becomes in control, and the solute loading is nearly independent of \( Q \); when \( \alpha \) becomes larger, say \( \alpha > 2.5 \), solute concentration would become close to its saturation concentration \( C_s \), and \( Q \) determines solute loading (that is, the solute loading is proportional to \( Q \)). When \( 0.5 < \alpha < 2.5 \), both \( Q \) and \( P \) are in control, a mixed control mechanism. The 20 years of measurements of mine drainage chemistry from the main waste rock piles at the Equity Silver mine, BC, Canada, are used to illustrate how to determine the two characteristic properties \( P \) and \( \alpha \), and how well they are able to describe the waste rock piles in terms of metal leaching.

1. Introduction

Waste rock piles are seen at mine sites, in particular, at open pit mines in virtually every country in the world. They contain leachable minerals that can be leached out by rain-water and/or snowmelt water for many decades to millennia, and the drainage waters from them pose serious hazards to the environment and to humans (Hudson-Edwards, 2016; Morin, 2011; Morin and Hutt, 2007; Weber et al., 2018; Hudson-Edwards et al., 2011; Liu et al., 2017). Leachable minerals can contain chemical elements such as arsenic, selenium, aluminum, lead, zinc, iron and copper, etc., and hence mine drainage can be toxic, corrosive, radioactive, or some combination of these. Large volumes of mine wastes have been accumulated by contemporary mining operations compared with the mine wastes previously produced because of the depletion of high-grade ores. As a consequence, the cost for collecting and treating the mine drainage has been increasing. It is not uncommon that it costs $2–$3 million annually to collect and treat mine drainage from a pile of many tens to hundreds of millions of tons of waste rocks,
a typical size at larger open-pit mine sites. Moreover, mine drainage treatment by the lime neutralization approach, the most widely used technology for treating mine drainage, leaves behind large amounts of sludge, whose storage poses a serious challenge due to its toxicity, physical and geochemical stability, and large volume.

Therefore, a practical challenge arises: how can waste rock piles be optimally configured to reduce solute loadings in mine drainage? Another practical challenge is how smaller-scale tests for predicting drainage chemistry, such as humidity cell testing, column-leach testing, and leach pad testing, can be designed so that test results can be accurately scaled up to the full field-size. To address these two challenges, one has to have a deep understanding of the metal leaching phenomenon in terms of what are the quantities/variables that can sufficiently describe the phenomenon and what are their relations, in particular, what are the most important characteristic properties of a waste rock pile in terms of metal leaching. Regarding the challenges there is little accumulation of knowledge.

For understanding the metal leaching phenomenon in a waste rock pile and predicting its long-term behavior, three categories of approaches have been developed and used. One category is to formulate the mathematical models (a set of differential equations plus boundary conditions and initial conditions) that govern mass transport through waste rock piles coupled with geochemical reactions, and then to solve the equations using finite element methods. According to the level of the sophistication of the mathematical models, the models could be conceptual models, or 1-D models, or 2-D models, or 3D pile-scale mega-models with hydrological, geochemical and even biological processes included. The advantages and disadvantages of this approach are outlined and/or reflected in research articles (Liu et al., 2017; Nordstrom and Nicholson, 2017; Williams and Diehl, 2014; Lefebvre et al., 2001a, 2001b; Molson et al., 2005; da Silva et al., 2009; Fala et al., 2005; Mayer, 2017; Ma et al., 2019). One notable advantage of this category is their ability to give relatively complete and detailed information with fine resolutions; one primary disadvantage is the difficulty of reducing their deviations from realities. The deviations are from both the assumptions made on the models and the difficult-to-attain properties that occur as the coefficients of the equations. For example, Darcy’s law-based models and even computational fluid dynamic (CFD)-based models are not able to capture the detailed physics of channeled flows, also called preferential flows or fingered flows, in which rainwater and snowmelt water infiltrate through waste rock piles. Nevertheless, the reality is that infiltrated water flows down through waste rock piles nearly always in the form of discrete individual flow channels (Li, 1999; Cesare et al., 2014; Wei et al., 2014; Annaka and Hanayama, 2010). The channeled flows at the scale of waste rock piles are extremely difficult, even if possible, to mathematically model and numerically simulate given the power of today’s computers. Because of this type of difficulty, the existing models have not been able to predict accurately drainage chemistry at the scale of waste rock piles.

The second category of approaches is to take sufficient amount of samples from a waste rock pile, and test the samples for their mineral dissolution rates using humidity cells, leach-columns, or leach pads, and then scale up the test results to the full field-size. This category has been widely applied in the mining industry, but metal leaching consultants have met foundational challenges and struggled with accurately scaling up the test results to the full field-size.

The third category is empirical, i.e., to curve-fit the historical monitoring data of drainage chemistry and discharge rates, and then use it for predicting drainage chemistry (Sarah et al., 2009; Villeneuve et al., 2017). Because the curve-fitting does not contain any information of the layouts of waste rock piles, this approach is not able to provide guidance for configuring future waste rock piles.

The objective of this research is to identify the most important characteristic properties of waste rock piles in terms of metal leaching, in particular, the characteristic properties that are able to indicate quantitatively the mineral dissolution potential and the rate control mechanisms of mineral dissolution from waste rock piles, including what variables/quantities constitute the characteristic properties. This objective is achieved through a simple and effective model.

Note that the characteristic properties of a phenomenon such as metal leaching from waste rock piles are nearly always associated with models, and it is by the models that the properties can be defined and determined. The models for defining/revealing characteristic properties are supposed to capture dominant factors only, and hence are simple but effective due to the fact that the number of dominant factors would be few. Hence the models for defining/revealing the characteristic properties of a phenomenon are rarely the 2-D and 3-D mega models that require large-scale numerical calculations such as finite element methods.

As a result, the nature of establishing the models for defining/revealing characteristic properties is both science and art. The model proposed here is of this category: being rapid and, in particular, allowing users to rapidly grasp the effects of the layouts of waste rock piles on mineral dissolution. We avoid mega-models such as 2-D or 3-D pile-scale models involving computational fluid dynamics coupled with geochemical reactions.

Another advantage of this model is that it reveals how the relevant variables conspire together to determine the top characteristic properties “solute production potential” and “rate control quotient” to affect solute concentrations and loadings. In addition, the model shows how the two characteristic properties are determined using site monitoring data, or smaller-scale testing such as column-leach testing.

<table>
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<tr>
<th>Nomenclature</th>
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2. Metal leaching from waste rock piles

The aqueous solubility of sulfide minerals contained in waste rocks is, generally speaking, much lower than the solubility of the associated sulfite oxidation products, also called as secondary minerals. For example, copper sulfide (CuS) is relatively insoluble in water compared to the aqueous solubility of CuSO₄, a product of copper sulfide oxidation. That is to say, for reaction products of copper sulfide contained in waste rocks to appear in mine drainage, two steps are essential: the first step is sulfide mineral oxidation by moisture and air (water and oxygen) with the catalyzing effects of bacteria (Colmer and Hinkle, 1947) releasing aqueous acidity, sulphate, metals, and other elements that may form secondary minerals; the second step is the dissolution of the oxidation reaction products, i.e., salts and acidity, into pore water that flows generally downward through individual flow channels in waste rock piles as mine drainage. To attain the whole picture of this phenomenon, both steps need to be well understood.

According to the relative rapidity of the sulfide mineral oxidation to the rapidity of the dissolution of sulfide oxidation products into pore water, the two steps can be analyzed separately without coupling them together by differentiating the three situations: (1) “sulfide mineral oxidation in control”, in which the rates of sulfide mineral oxidation are much slower than the rates of oxidation product dissolution; (2) “oxi-

The situation (1) is favored by low oxygen concentration and/or low moisture content inside waste rock piles, and/or frequent water-flushing through waste rock piles at high flow rates, while the situation (2) is favored by high oxygen concentration and high moisture content inside waste rock piles, and/or infrequent water-flushing, and/or water-flushing at very low flow rates. Waste rock piles tend to favor one of the situations depending upon local climate (precipitation, humidity and temperature), waste rock pile layout and pile size, and gas/water permeability. It also depends upon the history of waste rocks in the piles. For example, sulfide minerals on the surfaces of waste rocks could have been oxidized over time and significant amount of oxidation products (salts and acidity) could have been already accumulated on rock surfaces waiting to be leached.

One common characteristic of situation (2) is relatively high aqueous concentrations of sulfate, close to or at the solubility of gypsum. This is commonly observed in full-scale drainages at sulfide-bearing mine sites and in small-scale drainages from sulphide-bearing columns (e.g., Morin, 2011; Morin and Hutt, 2007; Weber et al., 2018). Therefore, this paper addresses common situation (2) without dealing with the other two situations. That is to say, this paper addresses the situation where there are significant amounts of salts and acidity on rock surfaces waiting to be leached by infiltrated rainwater and/or snowmelt water and/or groundwater.

3. Model development

3.1. Channeled flows in waste rock piles

According to our observations, it is through numerous separate flow channels, called channeled flow (Li, 1999; Cesare et al., 2014; Wei et al., 2014; Annaka and Hanayama, 2010), that rain water and/or snowmelt water predominantly flow through waste rock piles. The total number of active flow channels inside a waste rock pile could be steady within a certain range of water infiltration rates, but would occasionally increase when water infiltration rates increase beyond this range.

As the rocks within the flow channels become wet and actively rinsed, sulfide oxidation products on rock surfaces are gradually leached out. Exposed sulfide minerals on rocks outside the flow channels can continue oxidizing, because they are exposed to moisture and air, but are not washed and leached as there is little active water movement. That is to say, the rocks inside the flow channels contribute directly to the solute concentrations of mine drainage, while the rocks outside of the flow channels may contribute indirectly such as through diffusion and/or condensation migration.

Here we assume that the direct contribution in the channels outweighs the indirect contribution outside the channels, and hence the indirect contribution is neglected in this paper. Based on a case study of high-frequency monitoring of full-scale waste-rock drainage, the indirect contribution may be minor but can still be significant (Morin, 2017). This will be examined in a later paper.

Consider a single vertical flow channel in a waste rock pile. Here it is assumed that the pore volume within the flow channel does not change with time and is continuously flushed with water. It is also assumed that the pore water is well mixed horizontally so that solute concentrations within the flow channel are horizontally uniform, but the concentrations increase as the water flows downward along the vertical axis. In this way, the concentration C of a solute species is a function of vertical location x, expressed as C(x), which increases with x.

![Fig. 1. Dependence of solute concentration upon vertical Location.](Image)
At $x = 0$, $C(0) = 0$, meaning that fresh water enters into the top of the flow channel; when water travels downward it will progressively dissolve additional oxidation products (salts and acidity) on the rock surfaces into pore water, making concentration $C(x)$ increase as $x$ increases. At $x = \beta$, the bottom position of the flow channel (that is, the bottom position of the waste rock pile), concentration $C(\beta)$ achieves its largest value. Here it is assumed that the pore water volume does not increase due to dissolved minerals added into the pore water. This is reasonable, because rarely does total dissolved solids of waste-rock drainage significantly exceed 10 g/L, or roughly 1% of a liter.

### 3.2. The Noyes-Whitney equation

The Noyes–Whitney equation relates the rate of the dissolution of a solid particle to the properties of the solid and its dissolution medium. The relationship is given by:

$$\frac{d(m)}{dt} = A_p (C_s - C) \frac{D}{d}$$

where $\frac{d(m)}{dt} =$ mineral dissolution rate (mg/s), $m =$ the mass of a specific mineral dissolved into pore water (mg), $t =$ time (s), $A_p =$ the surface area of a solid particle (dm$^2$), $D =$ the diffusion coefficient of a mineral dissolved in pore water (dm$^2$/s), $C_s =$ the saturation concentration of a mineral in pore water (mg/L), $C =$ the concentration of a mineral in the bulk pore water (mg/L), and $d =$ diffusion layer thickness (dm).

To apply Eq. (1a) to the rocks that situate inside a flow channel, whose surfaces have specific secondary mineral particles with varying distribution densities, let us modify the format of Eq.(1a) as,

$$\frac{d(m)}{dt} = A_s k (C_s - C)$$

where $A_s =$ rock surface area that is submerged in pore water (dm$^2$), $k =$ effective/average dissolution coefficient of a specific mineral species on a rock surface, normalized to $A_s$ (dm/s).

Eq. (1b) shows that the dissolution rate of a specific secondary mineral species, $\frac{d(m)}{dt}$, is proportional to the difference between its saturation concentration and its concentration in the bulk pore water, and proportional to $k$, an average/effective dissolution coefficient of a specific mineral species on a rock surface that is submerged in pore water. As a material property, $k$ can have different values with respect to different mineral species, and in addition, $k$ is dependent on the distribution density of the mineral particles on a rock surface, becoming larger when the distribution density is higher.

### 3.3. The principle of conservation of mass

Now let us analyze by the mass conservation principle the change of solute concentration $C$ along a short vertical section $\Delta x$ within the flow channel as shown in Fig. 1. A mass balance is taken about the volume along the vertical axis $x$. Within a time period $\Delta t$, the amount of mineral, $S_{in}$, that enters the section from the top surface of the section, can be written as,

$$S_{in} = AV C(x,t) \Delta t$$

where $A$ is the cross-section area of the flow channel, $C(x,t)$ represents the concentration of the mineral at location $x$ and at time $t$, and $V$ is the vertical flow speed of pore water with its unit, with $AV$ being flow rate.
Here denote \( S_p \) as the amount of a secondary mineral species that is dissolved from the rock surfaces inside the section \( \Delta x \) within time period \( \Delta t \), and follow the Noyes-Whitney equation (Eq. (1b)) one can have,

\[
S_p = \lambda (A\Delta x) k [C - C(x, t)] \Delta t
\]

where \( \lambda (A\Delta x) \) represents the total surface area of the rocks inside the volume \( A\Delta x \), and \( \lambda \) is the ratio of the total rock surface area to the volume of the section \( \Delta x \), \((1/dm)\), called as surface area index, reflecting rock particle size, being larger when particle size is smaller.

At the bottom of the short section \( \Delta x \), the amount of the mineral species out, \( S_{out} \), within time period \( \Delta t \) can be written as,

\[
S_{out} = AV \left[ C(x, t) + \frac{\partial C}{\partial x} \Delta x \right] \Delta t
\]

In the practice of mine drainage monitoring, what is measured is not \( V \), the flow speed of pore water, but \( Q \), the flow rate of mine drainage, (L/s) in the channel.

One can get,

\[
\varepsilon A\Delta x \frac{\partial C}{\partial t} = AVC(x, t) \Delta t + \lambda (A\Delta x) k [C - C(x, t)] \Delta t - AV C(x, t) + \frac{\partial C}{\partial x} \Delta x
\]

which can be re-written as

\[
\frac{\partial C}{\partial t} = k \left[ C - C(x, t) \right] - V \frac{\partial C}{\partial x}
\]

Assuming steady state conditions, i.e., \( \varepsilon \frac{\partial C}{\partial t} = 0 \), one can have the following differential equation and boundary condition:

\[
\frac{dC(x)}{dx} = \frac{2k}{V} [C - C(x)]
\]

\[
C(0) = 0, \ x = 0
\]

This differential equation has the following solution,

\[
C(x) = C_i \left( 1 - e^{-\frac{2x}{V}} \right)
\]

At the bottom of the waste rock pile, i.e., \( x = \beta \), Eq. (9) can be written as,

\[
C(\beta) = C_i \left( 1 - e^{-\frac{2\beta}{V}} \right)
\]
measured at a low point on a mine drainage ditch. In a waste rock pile there are numerous flow channels, and they together contribute to flow rate Q. Designating \( \psi \) as the sum of the cross-section areas of the numerous flow channels in a waste rock pile, one has \( V = Q/\psi \).

Introducing \( V = Q/\psi \) to Eq. (10) yields,

\[
C(\beta) = C_s \left( 1 - e^{-\frac{k \lambda \psi}{Q}} \right)
\]  

(11)

Use \( P \) to represent \( \lambda \beta k \psi \), i.e., \( P = \lambda \beta k \psi \), and use \( \alpha \) to represent \( P/Q \), i.e., \( \alpha = P/Q \), Eq. (11) becomes,

\[
C(\beta) = C_s \left( 1 - e^{-\frac{P}{Q}} \right) = C_s \left( 1 - e^{-\alpha} \right)
\]  

(12)

Eq. (12) is the C-Q relation that has the two characteristic properties of a waste rock pile in terms of metal leaching, \( P \) and \( \alpha \), and will be discussed further in the next section.

4. Solute production potential \( P \)

The forms of \( P \) and \( \alpha \) in Eq. (12) contains valuable information. The four variables \( \lambda, \beta, k \), and \( \psi \) conspire together to determine one quantity \( P = \lambda \beta k \psi \). The product of the four variables together affecting solute concentration, \( C \), rather than independent influences from each of the four variables. Here we name \( P \) as solute production potential. \( P \) is a characteristic property of a waste rock pile in terms of mineral dissolution. By inspection of the dimensions of \( \lambda, \beta, k, \) and \( \psi, \) one can see

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**Fig. 5.** (a) Acidity concentration C vs flow rate Q (Bessemer). (b) Acidity concentration C vs flow rate Q (Bessemer).
that P has the same dimension as Q, i.e., L/s, Q being flushing rate. The larger the value of P is, the larger the potential for the associated mineral species to be dissolved in water and to end up in mine drainage at the bottom of a waste rock pile. P as the product of the four variables inform us that the effect of changing $\lambda$, i.e., rock surface area index, is exactly the same as that of changing $\beta$, i.e., rock pile depth. If we double $\lambda$, i.e., double rock surface area while keeping the other variables fixed, we affect solute concentration C precisely as we would if we doubled $\beta$, i.e., double rock pile depth, keeping the other variables constant. This provides a room for the configuration optimization of waste rock piles.

Using Eq. (12) solute production potential P of a waste rock pile can be conveniently determined by a set of mine drainage measurements, i.e., a set of C-Q measurements. Denote the set of measurements of C and Q as $(C_i, Q_i)$, $i = 1, 2, 3, \ldots, N$, and denote the associated solute concentrations given by Eq. (12) as $C_s(1 - \exp(-P/Q_i))$, $i = 1, 2, \ldots, N$. Define the following single variable optimization as,

$$\text{Minimize } \sum (C_i - C_s(1 - \exp(-P/Q_i)))^2 \text{ with respect to } P \quad (13)$$

By solving this optimization problem (13) the value of P can be determined.

5. Determination of k values by small scale tests

For small scale tests such as column leach tests or leach pad tests, $\lambda$,
\( \beta \) and \( \psi \) that are associated with the small scale tests can be determined conveniently. After a set of \( C-Q \) measurements from the small scale tests are available, the corresponding \( P \) values can be determined by Eq. (13). As long as \( P, \lambda, \beta, \) and \( \psi \) are available, one can calculate the associated \( k \) values from \( P = \lambda \beta k \psi \). When the same rocks as that inside the waste rock pile are tested, one can assume that the \( k \) values determined by the small scale tests would be the same as that of the rocks inside the pile. That is to say, the small scale tests can determine the three quantities of \( P, \lambda, \) and \( \psi \), and hence \( k \) values can be determined from \( P = \lambda \beta k \psi \). As long as the \( k \) values are available, one can determine the \( P \) values and \( \alpha \) values of the waste rock pile as long as \( \lambda, \beta, \) and \( \psi \) of the waste rock pile can be determined. Here it would not be a challenge to estimate \( \lambda, \beta \) of a waste rock pile because rock particle size distribution, pile volume and pile depth are often available. But it would be a challenge to determine \( \psi \), i.e., the sum of the cross-section areas of the flow channels inside a waste rock pile. It is our intention to give a separate report on how to determine \( \psi \).

6. Rate control quotient \( \alpha \): scaling law

An interesting thing of Eq. (12) is that \( P (P = k \lambda \beta \psi) \) and \( Q \) conspire together to form one quantity \( \alpha \) (\( \alpha = P/Q \)), and it is \( \alpha \) that dictates solute concentration \( C(\beta) \) instead of \( P \) and \( Q \) independently. Here \( \alpha \) is a dimensionless number, and is a ratio of \( P \) to \( Q \). The preservation of \( \alpha \) is crucial when upscaling: given any two waste rock piles, as long as their ranges of \( \alpha \) values are the same, the two waste rock piles share the same behavior in terms of the \( C-Q \) relation, whatever the individual values of \( P \) and \( Q \) of each waste rock pile are.

A practical implication of this scaling law is its guidance for designing and scaling up small scale tests of mineral dissolution to its full field-size. As long as \( \alpha \) values of a small-scale tests are in the same range of \( \alpha \) values of its full field-size, the \( C-Q \) relation given by the small-scale
tests would be the same as the C-Q relation of the full-size-field according the scaling law. Even if a small-scale tests and the full-size-field have different range of \( \alpha \) values, they can be still scaled to each other according to \( C(\beta) = C_s(1 - e^{-\alpha}) \).

7. Rate control quotient \( \alpha \): rate-control mechanisms

Here \( \alpha \) is not only a key parameter of the scaling law, but also the rate-control quotient. It quantitatively indicates the rate-control mechanisms, as shown in Fig. 2.

7.1. When \( 0 < \alpha < 0.5 \), solute production potential \( P \) (\( P = k\beta\lambda\phi \)) is in control

When \( 0 < \alpha < 0.5 \), \( 1 - e^{-\alpha} \) can be approximated as \( \alpha \), so the expression \( C(\beta) = C_s(1 - e^{-\alpha}) \) can be re-written as:

\[
C(\beta) \approx \alpha C_s \text{ or } C(\beta) \approx \frac{P C_i}{Q} \tag{14}
\]

Then the loading of the mineral species, \( T \), with its dimension as mg/s, can be expressed as:

\[
T = \frac{Q C(\beta)}{C_i} \tag{15}
\]
Eq. (15) shows that when $\alpha$ is small, say, $0 < \alpha < 0.5$, solute production potential $P$ ($P \equiv k\beta\psi$) is in control: the total loading is proportional to $P$, independent of flow rate $Q$; and Eq. (14) shows that solute concentration $C(\beta)$ is proportional to $P$, inversely proportional to $Q$.

When $\alpha > 2.5$, flow rate of mine drainage, $Q$, is in control: the total loading is proportional to $Q$, independent of solute production potential $P$, and the concentration $C(\beta)$ becomes almost constant being close to saturation concentration $C_s$.

When $0.5 < \alpha < 2.5$, both $Q$ and $P$ are in control of solute loading, a mixed control mechanism.

Then the loading of mineral species, $T$, can be expressed as:

$$T = QC(\beta) \approx QC_s \quad (17)$$

Eq. (17) shows that when $\alpha > 2.5$, flow rate of mine drainage, $Q$, is in control: the total loading is proportional to $Q$, independent of solute production potential $P$, and the concentration $C(\beta)$ becomes almost constant being close to saturation concentration $C_s$.

When $0.5 < \alpha < 2.5$, both $Q$ and $P$ contribute to $\alpha$ and neither can
be neglected. As a result, expression \( C(\beta) = C_s(1 - e^{-\alpha}) \) cannot be simplified in the above manners. It means that both flow rate \( Q \) and solute production potential \( P \) are in control, a mixed control mechanism. In this case, solute loading is

\[
T = QC(\beta) = QC_s(1-e^{-\alpha})
\]

(18)

8. Application to the waste rock piles of the Equity Silver mine, BC, Canada

8.1. Confirmation of the C-Q Relation \( C(\beta) = C_s(1 - e^{-\alpha}) \)

The C-Q relation \( C(\beta) = C_s(1 - e^{-\alpha}) \) shows the dependence of solute concentration \( C(\beta) \) upon \( \alpha \) (\( \alpha = P/Q = k\beta\lambda\psi/Q \)). The solute production potential \( P \) (\( P = k\beta\lambda\psi \)) is a characteristic property of a waste rock pile, and can be considered as a fixed quantity to a given waste rock pile. But \( Q \), the flow rate of mine drainage from the waste rock pile, depends upon the rate of the infiltration of rainwater and/or snowmelt water, and hence \( Q \) fluctuates a lot with time. The fluctuation of \( Q \) with time causes the fluctuation of \( C(\beta) \) with time. Here let us test the C-Q relation \( C(\beta) = C_s(1 - e^{-P/Q}) \) against a set of C-Q measurements from full-scale field monitoring data. For visualization, let us perform a coordinate transformation from C-Q relation to y-x relation: let \( y = \ln[1 - C(\beta)/C_s] \), and let \( x = 1/Q \). Under the y-x coordinate, the C-Q relation \( C(\beta) = C_s(1 - e^{-P/Q}) \) becomes the y-x relation as \( y = -Px \). The y-x relation is a straight line with its slope as \(-P \). The full-scale field measurements of C-Q are from the main waste rock pile.
and the Bessemer waste rock pile at the Equity Silver mine, BC, Canada, as shown in Figs. 3 and 4. The C-Q measurements spanned 20 years from 1998 to 2017. According to the Q-C relation, the full-scale data that is coordinate-transformed on the x-y coordinate should fall along a straight line $y = -Px$, passing through the origin point.

The main waste rock pile and the Bessemer waste rock pile sit at the lower part of Fig. 3. The mine drainage from all of the follow channels in the main waste rock pile ends up at C7 location on the acid mine drainage (ARD) ditch, which is shown by the blue line in Fig. 4; and the mine drainage from all of the follow channels in the Bessemer waste rock pile ends up at the Bessemer location on the ARD ditch, which is shown by the blue line in Fig. 4. At the two locations marked by C7 and Bessemer, respectively, the solute concentrations C and drainage-flow rates Q were measured over the past 20 years (1998–2017) at time intervals of 2 weeks. On C-Q coordinates and y-x coordinates as well, the dependences of the concentrations of ten different solutes on flow rates are plotted. To save space, only a few solutes are shown in Figs. 5–11. Not included are the months from October to May because flow rates in those months are either too low (winter seasons) or too high (it is known that in April and May there is a lot of groundwater coming from its nearby forest area flowing through the bottom part of the waste rock piles and ends up in the ARD ditches). It is speculated that from October to May the contribution from the ground water to the flow rates Q is significant compared with the amount of water flowing downward from the top of the waste rock piles to the bottom of the waste rock piles, as a consequence, those months deviate too much from
Table 1
The characteristic properties $P$ and $\alpha$ of the Bessemer waste rock pile, the Equity Silver Mine, BC, Canada, ($P \equiv k\beta\lambda\psi$, $\alpha \equiv P/Q$), determined from the C-Q Monitoring data from 1998 to 2017.

<table>
<thead>
<tr>
<th>Solute</th>
<th>Maximum concentration measured from 1998-2017 (mg/L)</th>
<th>$P$, Solute production potential (L/h)</th>
<th>$\alpha$, Rate control quotient range</th>
<th>Rate control mechanisms according to $\alpha$</th>
<th>$C_s$, Solute concentration (mg/L) by Eq.(12)</th>
<th>$T$, Solute loading (mg/s) by Eq.(17)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acidity</td>
<td>27100</td>
<td>2.79</td>
<td>0.3—1.1</td>
<td>Solute production zone and mixed zone</td>
<td>(0.26—0.67)$C_s$</td>
<td>(0.26—0.67)$QC_s$</td>
</tr>
<tr>
<td></td>
<td>27100</td>
<td>2.79</td>
<td>0.6—2.2</td>
<td>Mixed zone</td>
<td>(0.45—0.89)$C_s$</td>
<td>(0.23—0.45)$QC_s$</td>
</tr>
<tr>
<td>Al</td>
<td>2066</td>
<td>5.21</td>
<td>0.6—1.6</td>
<td>Mixed zone</td>
<td>(0.26—0.80)$C_s$</td>
<td>(0.26—0.80)$Q^C_s$</td>
</tr>
<tr>
<td></td>
<td>2066</td>
<td>5.21</td>
<td>0.6—3.2</td>
<td>Mixed zone and saturation zone</td>
<td>(0.45—0.96)$C_s$</td>
<td>(0.23—0.48)$Q^C_s$</td>
</tr>
<tr>
<td>Mg</td>
<td>4240</td>
<td>4.8</td>
<td>0.6—1.8</td>
<td>Solute production zone and mixed zone</td>
<td>(0.26—0.83)$C_s$</td>
<td>(0.26—0.83)$Q^C_s$</td>
</tr>
<tr>
<td></td>
<td>4240</td>
<td>4.8</td>
<td>0.6—3.6</td>
<td>Mixed zone and saturation zone</td>
<td>(0.45—0.97)$C_s$</td>
<td>(0.23—0.48)$Q^C_s$</td>
</tr>
<tr>
<td>Mn</td>
<td>654</td>
<td>4.85</td>
<td>0.3—1.1</td>
<td>Solute production zone and mixed zone</td>
<td>(0.26—0.67)$C_s$</td>
<td>(0.26—0.67)$Q^C_s$</td>
</tr>
<tr>
<td></td>
<td>654</td>
<td>4.85</td>
<td>0.6—2.2</td>
<td>Mixed zone</td>
<td>(0.45—0.89)$C_s$</td>
<td>(0.23—0.45)$Q^C_s$</td>
</tr>
<tr>
<td>Fe</td>
<td>1779</td>
<td>3.55</td>
<td>0.3—1.4</td>
<td>Solute production zone and mixed zone</td>
<td>(0.26—0.76)$C_s$</td>
<td>(0.26—0.76)$Q^C_s$</td>
</tr>
<tr>
<td></td>
<td>1779</td>
<td>3.55</td>
<td>0.6—2.8</td>
<td>Mixed zone and saturation zone</td>
<td>(0.45—0.94)$C_s$</td>
<td>(0.23—0.47)$Q^C_s$</td>
</tr>
<tr>
<td>Ni</td>
<td>16.7</td>
<td>5.68</td>
<td>0.5—2.1</td>
<td>Mixed zone and saturation zone</td>
<td>(0.39—0.88)$C_s$</td>
<td>(0.39—0.88)$Q^C_s$</td>
</tr>
<tr>
<td></td>
<td>16.7</td>
<td>5.68</td>
<td>1.0—4.2</td>
<td>Mixed zone and saturation zone</td>
<td>(0.63—0.99)$C_s$</td>
<td>(0.32—0.50)$Q^C_s$</td>
</tr>
<tr>
<td>Zn</td>
<td>700.8</td>
<td>4.28</td>
<td>0.3—1.7</td>
<td>Solute production zone and mixed zone</td>
<td>(0.26—0.82)$C_s$</td>
<td>(0.26—0.82)$Q^C_s$</td>
</tr>
<tr>
<td></td>
<td>700.8</td>
<td>4.28</td>
<td>0.6—3.4</td>
<td>Mixed zone and saturation zone</td>
<td>(0.45—0.97)$C_s$</td>
<td>(0.23—0.49)$Q^C_s$</td>
</tr>
</tbody>
</table>

Blue color: the current configuration of the Bessemer waste rock pile.
Red color: the alternative configuration of the Bessemer waste rock pile described above.
Table 2
The characteristic properties $P$ and $\alpha$ of the main waste rock pile at C7 location, the Equity Silver Mine, BC, Canada, ($P = k_0^{\beta \lambda \psi}$, $\alpha = P/Q$), determined from the C-Q monitoring data from 1998 to 2017.

<table>
<thead>
<tr>
<th>Solute</th>
<th>Maximum concentration measured from 1998-2017 as saturation (mg/L)</th>
<th>$P$, Solute production potential (L/s)</th>
<th>$\alpha$, Rate control quotient range</th>
<th>Rate control mechanisms according to $\alpha$</th>
<th>$C$, Solute concentration (mg/L) by Eq. (12) $C = C_r(1-\epsilon^{-\alpha})$</th>
<th>$T$, Solute loading (mg/s) by Eq. (17) $T = QC_r(1-\epsilon^{-\alpha})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acidity</td>
<td>37763</td>
<td>2.47</td>
<td>0.1—0.7</td>
<td>Solute production zone and mixed zone</td>
<td>(0.10—0.50)$C_r$</td>
<td>(0.10—0.50)QC_r</td>
</tr>
<tr>
<td></td>
<td>37763</td>
<td>2.47</td>
<td>0.2—1.4</td>
<td>Solute production zone and mixed zone</td>
<td>(0.18—0.73)$C_r$</td>
<td>(0.09—0.37)QC_r</td>
</tr>
<tr>
<td>Al</td>
<td>3545</td>
<td>2.66</td>
<td>0.1—0.8</td>
<td>Solute production zone and mixed zone</td>
<td>(0.63—0.55)$C_r$</td>
<td>(0.63—0.55)QC_r</td>
</tr>
<tr>
<td></td>
<td>3545</td>
<td>2.66</td>
<td>0.2—1.6</td>
<td>Solute production zone and mixed zone</td>
<td>(0.18—0.80)$C_r$</td>
<td>(0.09—0.40)QC_r</td>
</tr>
<tr>
<td>Mg</td>
<td>2493</td>
<td>4.6</td>
<td>0.05—1.4</td>
<td>Solute production zone and mixed zone</td>
<td>(0.05—0.75)$C_r$</td>
<td>(0.05—0.75)QC_r</td>
</tr>
<tr>
<td></td>
<td>2493</td>
<td>4.6</td>
<td>0.1—2.8</td>
<td>All of the three zones</td>
<td>(0.10—0.94)$C_r$</td>
<td>(0.05—0.47)QC_r</td>
</tr>
<tr>
<td>Mn</td>
<td>336.8</td>
<td>4.11</td>
<td>0.15—1.2</td>
<td>Solute production zone and mixed zone</td>
<td>(0.14—0.70)$C_r$</td>
<td>(0.14—0.70)QC_r</td>
</tr>
<tr>
<td></td>
<td>336.8</td>
<td>4.11</td>
<td>0.3—2.4</td>
<td>Solute production zone and mixed zone</td>
<td>(0.25—0.91)$C_r$</td>
<td>(0.13—0.46)QC_r</td>
</tr>
<tr>
<td>Fe</td>
<td>6920</td>
<td>1.48</td>
<td>0.05—0.45</td>
<td>Solute production zone</td>
<td>(0.05—0.36)$C_r$</td>
<td>(0.05—0.36)QC_r</td>
</tr>
<tr>
<td></td>
<td>6920</td>
<td>1.48</td>
<td>0.1—0.9</td>
<td>Solute production zone and mixed zone</td>
<td>(0.10—0.59)$C_r$</td>
<td>(0.05—0.30)QC_r</td>
</tr>
<tr>
<td>Zn</td>
<td>368.0</td>
<td>2.61</td>
<td>0.1—0.9</td>
<td>Solute production zone and mixed zone</td>
<td>(0.10—0.59)$C_r$</td>
<td>(0.10—0.59)QC_r</td>
</tr>
<tr>
<td></td>
<td>368.0</td>
<td>2.61</td>
<td>0.2—1.8</td>
<td>Solute production zone and mixed zone</td>
<td>(0.18—0.83)$C_r$</td>
<td>(0.09—0.42)QC_r</td>
</tr>
<tr>
<td>Cu</td>
<td>346.1</td>
<td>2.27</td>
<td>0.05—0.8</td>
<td>Solute production zone and mixed zone</td>
<td>(0.05—0.55)$C_r$</td>
<td>(0.05—0.55)QC_r</td>
</tr>
<tr>
<td></td>
<td>346.1</td>
<td>2.27</td>
<td>0.1—1.6</td>
<td>Solute production zone and mixed zone</td>
<td>(0.10—0.88)$C_r$</td>
<td>(0.05—0.46)QC_r</td>
</tr>
</tbody>
</table>
the model, which addresses channelled flows only from the top of a waste rock pile to its bottom. Other elements such as Se and As had fewer measurements during the monitoring period and thus were not assessed here. The overall trends of the data shown in Figs. 5–11 seem quite supportive of the C-Q relation \( C(\beta) = C_0(1 - e^{-P/Q}) \) in that their overall trends are in agreement with straight lines \( y = -Px \). The correlation coefficients are in the range of 0.65–0.80.

8.2. \( P \) and \( \alpha \), the two characteristic properties of the waste rock piles

The slopes of the straight lines in Figs. 5–11 have clear physical meaning, i.e., they are exactly the values of \( P (P = -k\beta\lambda\psi) \). The range of the data points in the \( y \)-coordinate has also clear physical meaning, i.e., it is exactly the range of \( \alpha \) values, representing rate control mechanisms. Note that the rate-control mechanisms depend upon saturation concentration \( C_s \). The saturation concentrations used in Figs. 5–11 were the maximum ones among the concentrations measured from 1998 to 2017.

When \( C_s \) is available and \( P (P = k\beta\lambda\psi) \) is determined by Eq. (13), the range of \( \alpha \) can be calculated based on the range of \( Q \), the flow rate of mine drainage measured. Table 1 lists the values of \( P \) and \( \alpha \) corresponding to the Bessemer waste rock pile, and Table 2 lists the values of \( P \) and \( \alpha \) corresponding to the main waste rock pile. They also list rate control mechanisms according to the range of \( \alpha \) values, which are the range of the data points on the \( y \)-coordinate on Figs. 5–11. The last two columns show how solute concentrations \( C \) and solute loadings \( T \) depend on flow rate \( Q \) according to Eqs. (12) and (18).

The two characteristic properties \( P \) and \( \alpha \) allow us to see the effect of changing the configuration of the waste rock piles on rate control mechanisms, solute concentration \( C \) and solute loadings \( T \). Let us illustrate this. Consider a 50% reduction of the footprint on each of the two waste rock piles and an increase in their height by 50% each keeping the other configuration variables such as rock sizes unchanged. This change would lead to a reduction of \( \psi \) by 50% (\( \psi \) is the sum of the cross-section areas of flow channels in a waste rock pile) and an increase of \( \beta \) to the 2 times of the original height. Hence solute production potential \( P (P = k\beta\lambda\psi) \) would not change at all due to the fact that the effect of doubling \( \beta \) is canceled out by a reduction of \( \psi \) by 50%. But this configuration change would lead to 50% reduction of \( Q \), flow rate of mine drainage, due to 50% reduction of the footprint. Hence, rate control quotient \( \alpha (\alpha = P/Q) \) would be doubled. The increase of \( \alpha \) by 100% implies that rate control mechanism would shift towards to the saturation zone, as shown in Fig. 2, and would lead to a change in solute concentration \( C \) and solute loading \( T \) in the way as Eqs. (12) and (18). These changes induced by the configuration change, as listed in Tables 1 and 2, show that the configuration change of the waste rock piles as described above could lead to the reductions of solute concentrations and solute loadings, and can shift rate control mechanisms towards to the solute saturation zone, meaning reduced solute loadings.

9. Concluding remarks

This paper reveals the two characteristic properties of waste rock piles: solute production potential \( P (P = k\beta\lambda\psi) \) and rate control quotient \( \alpha (\alpha = P/Q) \), for characterizing waste rock piles in terms of mineral dissolution. The characteristic properties are defined by a simple and effective model that relates solute concentration \( C \) with flow rate \( Q \). The model gives an explicit C-Q relation as \( C = C_0(1 - e^{-P/Q}) \). This relation is in good agreement with the 20 years of monitoring data at the Equity Silver mine. A few interesting things shown by the C-Q relation include: (1) solute concentration \( C \) is dependent upon one quantity \( \alpha \) alone rather than the five quantities \( k, \beta, \lambda, \psi \) and \( Q \) separately; (2) the quantity \( \alpha \) is a ratio of \( P \) to \( Q \), where \( P \) represents solute production potential, and \( Q \) represents water-flushing rate; \( \alpha \) is a pure number, a non-dimensional quantity; (3) in terms of mineral dissolution, the four variables of \( k, \beta, \lambda, \psi \), combine together to form one quantity \( P (P = k\beta\lambda\psi) \), which determines the mineral dissolution potential of a waste rock pile instead of individual quantities of \( k, \beta, \lambda, \psi \); in another words, the product \( k\beta\lambda\psi \) is the characteristic property of a waste rock pile instead of the four individual variables; (4) the quantity \( \alpha \) is the scaling law, by which smaller-scale test results can be scaled up to its full-scale field; (5) the quantity \( \alpha \) is an indicator of the rate-control mechanism, quantitatively indicating rate-control mechanisms. When \( \alpha \) is small, say \( \alpha < 0.5 \), solute production potential \( P \) determines solute loadings; when \( \alpha \) is large, say \( \alpha > 2.5 \), flow rate \( Q \) determines solute loadings. When \( 0.5 < \alpha < 2.5 \), both \( P \) and \( Q \) exert control, a mixed control mechanism. The rate-control mechanisms are needed for determining optimal configurations for waste rock piles so that solute loadings can be reduced. The authors believe that the concept here would be effective and quantitative for understanding how minerals are leached out from waste rock piles.

While applying the C-Q relation given here, one needs to know its underlying assumptions: (1) mineral dissolution rate from rock surfaces follows the Noyes-Whitney equation as shown in Eq. (1b); (2) solute concentration is uniformly distributed horizontally; (3) channelled flow is assumed to be the major form in which infiltrated water flows through a waste rock pile; (4) the effect of water storage inside rocks and pores on flow rates is neglected; (5) infiltrated water comes mainly from the top of a waste rock pile instead of flowing through a reservoir at the bottom part of a waste rock pile. The assumption (4) could be the reason why this model does not work well for those months when infiltration rates become very low. The assumption (5) could be the reason why the model does not work well for those months when ground water contributes considerably to discharge rates in the ARD ditches at the Equity Silver mine. Another assumption is that the model is mainly for the situation of “minerals-dissolution in control”. That is to say, the model might not work well if the rates of sulphide oxidation in a waste rock pile is slower then the dissolution rates of sulphide oxidation products into pore water. Regarding the Noyes-Whitney equation, the mineral dissolution rate equation, we do not doubt its wide range of applicability in mineral dissolution of from rock surfaces to pore water. A difficulty in use of it would be quantifying the effective/average dissolution coefficient \( k \). However, \( k \) values can be determined as described above.

References


Blue color: the current configuration of the main waste rock pile.
Red color: the alternative configuration of the main waste rock pile described above.


Williams, R.D., Diehl, S.F., 2014. Techniques for Predicting Metal Mining Influenced Water. 5 Society for Mining, Metallurgy & Exploration, Colorado, USA.